## 1. Details of Module and its structure

| Module Detail |  |
| :---: | :---: |
| Subject Name | Physics |
| Course Name | Physics 02 (Physics Part-2, Class XI) |
| Module Name/Title | Unit 7, Module 13, Thermal Expansion Chapter 11, Thermal Properties of Matter |
| Module Id | keph_201102_eContent |
| Pre-requisites | Students should have knowledge of molecular arrangement inside materials, temperature, temperature scales, thermometers |
| Objectives | After going through this module, the learners will be able to: <br> - Understand Thermal expansion as expansion or contraction in dimensions of a material due to addition or removal of heat energy. <br> - Explain, the effect of heating a bi-metallic strip and apply the same in real life <br> - Know the salient features of expansion of solids, liquids and gases <br> - Design and use an activity to study the changes in level of liquid in a container on heating and interpret the result <br> - Know about the anomalous expansion of water and its importance in nature |
| Keywords | Thermal expansion, Thermal stress, Expansions of gases |

## 2. Development Team

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## 1. UNIT SYLLABUS

UNIT 7:
PROPERTIES OF B ULK MATTER:
24 periods
Chapter-9: Mechanical Properties of Solids:
Elastic behaviour, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear, modulus of rigidity, Poisson's ratio, elastic energy.

Chapter-10: Mechanical Properties of Fluids:
Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes). Effect of gravity on fluid pressure. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its applications. Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise

## Chapter-11: Thermal Properties of Matter:

Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity; $\mathrm{Cp}, \mathrm{Cv}$ - calorimetry; change of state - latent heat capacity. Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wien's displacement Law, Stefan's law, Greenhouse effect.

| Module 1 | - Forces between atoms and molecules making up the bulk matter <br> - Reasons to believe that intermolecular and interatomic forces exist <br> - Overview of unit <br> - State of matter <br> - Study of a few selected properties of matter <br> - Study of elastic behaviour of solids <br> - Stationary fluid property: pressure and viscosity <br> - Stationary liquid property: surface tension <br> - Properties of Flowing fluids <br> - Effect of heat on matter |
| :---: | :---: |
| Module 2 | - Idea of deformation by external force <br> - Elastic nature of materials <br> - Elastic behaviour <br> - Plastic behaviour <br> - Tensile stress <br> - Longitudinal Stress and longitudinal strain <br> - Relation between stress and strain <br> - Hooke's law <br> - Young's modulus of elasticity ' $Y$ ' |
| Module 3 | - Searle's apparatus <br> - Experiment to determine Young's modulus of the material of a wire in the laboratory <br> - What do we learn from the experiment? |
| Module 4 | - Volumetric strain <br> - Volumetric stress <br> - Hydraulic stress <br> - Bulk modulus K <br> - Fish, aquatic life on seabed, deep sea diver suits and submarines |
| Module 5 | - Shear strain <br> - Shear stress <br> - Modulus of Rigidity G <br> - Poisson's ratio <br> - Elastic energy |

\(\left.\begin{array}{|l|ll|}\hline \& \bullet \& To study the effect of load on depression of a suitably <br>

clamped meter scale loaded at i)its ends ii)in the middle\end{array}\right]\)|  | $\bullet$ |
| :--- | :--- |
| Module 6 | Height of sand heaps, height of mountains |

\(\left.\begin{array}{|l|l|l|}\hline \& \bullet \& Application of surface tension to drops, bubbles <br>
\bullet \bullet \& Capillarity <br>
\bullet \& Determination of surface tension of water by capillary rise <br>

method in the laboratory\end{array}\right]\)|  | $\bullet$ To study the effect of detergent on surface tension of water |
| :--- | :--- |
|  |  |
|  | through observations on capillary rise. |


|  | - Conduction, convection, radiation <br> - Coefficient of thermal conductivity <br> - Convection |
| :---: | :---: |
| Module 17 | - Black body <br> - Black body radiation <br> - Wien's displacement law <br> - Stefan's law <br> - Newton's law of cooling, <br> - To study the temperature, time relation for a hot body by plotting its cooling curve <br> - To study the factors affecting the rate of loss of heat of a liquid |

## MODULE 13

## 3. WORDS YOU MUST KNOW

Heat: A form of energy associated with the movement of atoms and molecules in any material. The higher the temperature of a material, the faster the atoms are moving, and hence the greater the amount of energy present as heat.

Temperature: A measure of hotness and coldness in degrees.
Temperature scale: in which measurements are amounts that are more or less than a reference amount. In the Celsius scale, for example, the reference amount is set as the freezing point of water, or zero.

Celsius: denoting a scale of temperature on which water freezes at $0^{\circ}$ and boils at $100^{\circ}$ under standard conditions.

Kelvin: The SI base unit of thermodynamic temperature, equal in magnitude to the degree Celsius.

Thermometer: A device used for measuring temperature.
Thermostat: A device that automatically controls heating or cooling equipment in such a way as to maintain a temperature at a constant level or within a specified range.

Internal Molecular arrangement: In a crystalline solid the atoms / molecules are arranged in a pattern. The pattern or lattice is repeated in long-range order. In amorphous solids there is no regular pattern
'Potential energy-intermolecular separation' graph:

$\mathrm{R}_{0}$ corresponds to the separation between atoms /molecules at minimum potential energy

## 4. INTRODUCTION

It is our common experience that, most substances expand on heating and contract on cooling. We can cite several examples, from our daily life, in support of this observation. You may have observed that sometimes sealed bottles with metallic lids are so tightly screwed that one has to put the bottle upside down, with the lid in hot water for some time to open the lid. This allows the metallic cover to expand, thereby loosening it so that it can be unscrewed easily.

https://sc02.alicdn.com/kf/HTB1QFmOMpXXXXXPaXXXq6xXFXXXa/Eco-Friendly-food-grade-food-packing-cheap.jpg 350x350.jpg

In case of liquids, you may have observed that mercury, in a thermometer rises, when the thermometer is put in warm water. If we take out the thermometer from the warm water the level of mercury falls again. The thermometer used to measure body temperature works the same way
(It has a kink which does not allow mercury to fall back into the bulb. the mercury is brought back into the bulb by jerking it carefully)

Similarly, in the case of gases, a balloon, partially inflated in a cool room, may expand to full size when placed in warm water. On the other hand, a fully inflated balloon, when put in cold water would start shrinking due to contraction of the air inside.

Heat expansion may be viewed as a result of the increase of kinetic energy of the molecules. As their movement increases, they bump into each other more often and the average separation, between the molecules increases. The material (slightly) expands on heating. This is generally true for most of the materials.

We need to take note of thermal expansion in a variety of practical situations. Some of these are listed below:
(i) When a dental cavity is filled, the filling material must have the same thermal expansion properties as the surrounding teeth. Consuming cold / hot coffee would be very painful, if it were not so.
(ii) When an aircraft flies faster than the speed of the sound, the temperature increase causes the cabin windows to become noticeably warm to the touch. The length of the aircraft also increases.
(iii) The time period of a pendulum wall clock will increase, if environment temperature increases.
(iv) A power transmission line (electricity wires) on a hot day will be droopy, but on a cold day it would be tight. This is because metals expand when heated.

When matter is heated without any change in its state, it usually expands.
(However, some rubber like substances contract on heating because the transverse vibration of the atoms of substances dominate over their longitudinal vibrations which are responsible for expansion.)

## 5. CAUSE OF THERMAL EXPANSION- MOLECULAR EXPLAINATION

We know that, at any temperature, the atoms in a solid vibrate about their mean positions. As the temperature increases the amplitude of vibration increases.

However, this does not quite explain why the distance between equilibrium positions of two adjacent atoms should increase with increase in temperature.

We can understand thermal expansion using the curve, showing the dependence of potential energy on intermolecular separation. It has an asymmetrical nature.

With increase in temperature, the amplitude of vibration, and hence the energy of the atoms, increases.

The asymmetrical nature of the potential energy curve implies that this increase in energy must be accompanied by an increase in the average separation between the molecules. It is this increase in average separation that causes the material, as a whole, to undergo an expansion.


## Potential energy due to inter-atomic forces vs. interatomic separation

https://upload.wikimedia.org/wikipedia/commons/thumb/7/7a/Morse-potential.png/649px-Morse-potential.png

Observe the potential energy vs interatomic separation graph. The curve is asymmetric about its minimum value. Around the minimum value point, the attractive force part of the curve rises more slowly as compared to the repulsive part of the curve.

At any temperature, the energy of the molecules makes them vibrate with unequal displacement about the mean position. The vibrations, therefore, cease to be harmonic vibrations with equal displacement about the mean position. This results in an expansion of the material.

## THINK ABOUT THESE

$\checkmark$ Will there be a difference in the thermal expansion of material at the same temperature?
$\checkmark$ What is that maximum length up to which a material can expand when exposed to heat?
$\checkmark$ Does expansion depend upon how the material is being heated?
$\checkmark$ Will it depend upon shape of the material?
$\checkmark$ Will expansion depend upon the method employed to heat the system?

## SALIENT FEATURES OF EXPANSION IN SOLIDS

- Unlike gases or liquids, solid materials tend to keep their shape even after undergoing thermal expansion.
- Thermal expansion generally decreases with increasing bond energy. This also has an effect on the melting point of solids; so, high melting point materials are more likely to have lower thermal expansion
- The thermal expansion of amorphous materials is higher compared to that of crystals.
- The thermal energy increases with temperature; this increases the average separation of the atoms and thus the size of a solid sample.
- A change in the temperature of a solid body causes change in its dimensions. The increase, in the dimensions of a body due to the increase in its temperature, is called thermal expansion.


## TYPES OF THERMAL EXPANSION IN SOLIDS -

The expansion in length is called linear expansion. The expansion in area is called areal or superficial expansion. The expansion in volume is called volume expansion.

$\frac{\Delta l}{l}=a_{1} \Delta T \quad \frac{\Delta A}{A}=2 a_{1} \Delta T$
(a) Linear expansion
(b) Area expansion
(c) Volume expansion

Let us now consider each of these types of thermal expansion, separately.

## 6. LINEAR EXPANSION

When calculating thermal expansion, it is necessary to consider whether the body is free to expand or is constrained. If the body is free to expand, the expansion, or strain, resulting from an increase in temperature, can be simply calculated by using the applicable coefficient of thermal expansion.

If the body is constrained so that it cannot expand, then internal stress will be caused (or changed) by a change in temperature. This stress can be calculated by considering the strain that would occur if the body were free to expand and the stress required to reduce that strain to zero, through the stress/strain relationship characterised by relevant the elastic or Young's modulus, of the material.

The solid would expand in all directions (3D), but if its length is very much more than its other dimensions, it would be, its changes in length that would be primarily noticeable.

For example, if the object is in the form of a long rod, wire, narrow strip, rail girder etc, we need to talk mainly only about 'changes in length'.
when the length of a solid increases on heating, the thermal expansion is called linear expansion.
Before heating
After heating
Suppose a solid rod, of length $\mathrm{L}_{0}$, is heated through a temperature $\Delta T$, and its final length L . It is found from experiment that the change in its length $\left(\Delta L\left(=L-L_{0}\right)\right)$,
(i) $\Delta L \propto \Delta T$
and, (ii) $\Delta \mathrm{L} \propto \mathrm{L}_{0}$
Hence, for small change in temperature, $\Delta \mathrm{T}$, the fractional change in length, $\frac{\Delta L}{L_{0}}$, is directly proportional to $\Delta \mathrm{T}$.

Combining (i) and (ii)

$$
\begin{gathered}
\Delta \mathrm{L} \propto \Delta \mathrm{~T} \\
\Delta \mathrm{~L}=\propto \mathrm{L}_{0} \Delta \mathrm{~T}
\end{gathered}
$$

The proportionality constant $\alpha$ is called coefficient of linear expansion.

$$
\alpha=\frac{\Delta \mathbf{L}}{\mathbf{L}_{\mathbf{0}} \Delta \mathbf{T}}
$$

Hence the coefficient of linear expansion of the material of a solid rod is defined as the increase in length per unit original length per degree rise in its temperature.

The increased length of the rod is

$$
\mathbf{L}=\mathbf{L}_{\mathbf{0}}[\mathbf{1}+\boldsymbol{\alpha} \Delta \mathbf{T}]
$$

The S.I. unit of alpha $\alpha$ coefficient of linear expansion is ${ }^{0} \mathrm{C}^{-1}$ or $\mathrm{K}^{-1}$
(Watch the video to see an experiment showing expansion and contraction of a metal strip on heating and cooling.)


Metal expansion and contraction experiment


An experiment to demonotrate that metal eppuds when heated and cortract when cooled
hat when tap wate is poured on l . The rotation ot the white straw amplifes the expansion
swow mone

## https://www.youtube.com/watch?v=kDktat01G_E

## THINK ABOUT THESE

- Does linear expansion depend upon material and the change in its temperature
- Is there a relation between linear strain, increase in length and original length?
- Will coefficient of linear expansion depend upon the nature of material?
- Can we somehow prevent the body to expand when the temperature rises?
- Can there be a linear expansion in liquid?
- Is the coefficient of linear expansion for a material a constant at all temperatures?


## 7. BIMETALLIC STRIP

A useful device based on unequal expansion of materials when subjected to the same change of temperature.


## "hyperphysics"

A bi-metallic strip is a flat rectangular arrangement, with two different metal strips joined together back to back. It can be used to work as a thermostat.

A thermostat is a devise which controls the temperature of a system (electric iron, refrigerator, air conditioners, geysers, water heating kettles etc) and performs actions so that the system's temperature is maintained near a desired value.

## https://encrypted- <br> tbn0.gstatic.com/images?q=tbn\%3AANd9GcTfeX5VOCycN2wJVx2YKvo8YN9KGIRUK2 kYtJbmrKzs2noKRvOM

The figure shows a brass and a steel strip riveted together.
Brass has a larger value for its coefficient of linear expansion than steel. When the temperature rises, the brass strip will expand more than the steel one and thus the strip will bend downward. This can be used in an electrically heated system, to make the thermostat to turn on a device, using a suitable electrical circuit. When the temperature goes below the set value, the brass rod contracts more than the steel one and it bends restart the electrical heating circuit. This makes the heating to be turned off.

The coefficient of linear expansion is a characteristic of the material of the rod. Some typical average values of the coefficient of linear expansion for some materials, in the temperature range $0^{\circ} \mathrm{C}$ to $100^{\circ} \mathrm{C}$, are given in the table below

Values of coefficient of linear expansion for some substances

| Materials | $\alpha_{1}\left(10^{-5} \mathrm{~K}^{-1}\right)$ |
| :--- | :--- |
| Aluminium | 2.5 |
| Brass | 1.8 |
| Iron | 1.2 |
| Copper | 1.7 |
| Silver | 1.9 |
| Gold | 1.4 |
| Glass (pyrex) | 0.32 |
| Lead | 0.29 |

From this Table, compare the value of $\alpha$ for glass and copper. We find that copper expands about five times more than glass for the same rise in temperature. Normally, metals expand more and have relatively high values of $\alpha$.

## EXAMPLE

A thin copper rod is 5 m long at $15^{\circ} \mathrm{C}$ on a winter day. Calculate the increase in its length on a hot summer day when the temperature rises to $40{ }^{\circ} \mathrm{C}$.

SOLUTION

$$
\begin{gathered}
\Delta L=\propto L_{0} \Delta T \\
\therefore \Delta L=\left(1.7 \times 10^{-5} \times 5 \times(40-15)\right) \mathrm{m} \\
\therefore \Delta L=212.5 \times 10^{-5} \mathrm{~m}=2.12 \mathrm{~mm}
\end{gathered}
$$

## EXAMPLE

A steel tape, 1m long, is correctly calibrated for a temperature of $27.0^{\circ} \mathrm{C}$. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is $45.0^{\circ} \mathrm{C}$. What is the actual length of the steel rod on that day?

What is the length of the same steel rod on a day when the temperature is $27.0^{\circ} \mathbf{C}$ ?
Coefficient of linear expansion of steel $=1.20 \times 10^{-5} \mathrm{~K}^{-1}$

## SOLUTION

Length of steel tape at $27^{\circ} \mathrm{C}$ is 100 cm i.e. $\mathrm{L}_{0}=100 \mathrm{~cm}, \mathrm{~T}_{1}=27^{\circ} \mathrm{C}$
Length of steel tape at $45^{\circ} \mathrm{C}$ is $\mathrm{L}=\mathrm{L}_{0}+\Delta \mathrm{L}$

$$
=100+1.20 \times 10^{-5} \times 100(45-27)=100.0216 \mathrm{~cm}
$$

Length of 1 cm mark at $27^{\circ} \mathrm{C}$ on this scale, at $45^{\circ} \mathrm{C}=100.0216 / 100$
Length of 63 cm measured by this tape at $45^{\circ} \mathrm{C}$ will be $=\frac{100.0216}{100} \times 63=63.0136 \mathrm{~cm}$
Length of same steel rod on a day when the temperature is $27^{\circ} \mathrm{C}=63 \times 1=63 \mathrm{~cm}$

## EXAMPLE

A brass wire 1.8 m long at $27^{\circ} \mathrm{C}$ is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of $-39^{\circ} \mathrm{C}$, what is the tension developed in the wire, if its diameter is $\mathbf{2 . 0} \mathbf{~ m m}$ ? Co-efficient of linear expansion of brass $=\mathbf{2 . 0} \times 10^{-5} \mathrm{~K}^{\mathbf{- 1}}$; Young's modulus of brass $=0.91 \times 10^{11} \mathrm{~Pa}$.

## SOLUTION

Given: $\mathrm{L}=1.8 \mathrm{~m}, \mathrm{~T}_{1}=27^{\circ} \mathrm{C}, \mathrm{T}_{2}=-39^{\circ} \mathrm{C}$, tension developed in the wire, $\mathrm{F}=$ ?

$$
\begin{aligned}
& \mathrm{r}=1 \mathrm{~mm}=10^{-3} \mathrm{~m}, \quad \alpha=2.0 \times 10^{-5} \mathrm{~K}^{-1}, \quad \mathrm{Y}=0.91 \times 10^{11} \mathrm{~Pa} \\
& \mathrm{Y}=\frac{F L}{a \Delta L} \Rightarrow \Delta L=\frac{F L}{a Y} \\
& \alpha \mathrm{~L} \Delta \mathrm{~T}=\frac{F L}{a Y} \Rightarrow \alpha \Delta \mathrm{~T}=\frac{F}{a Y} \\
& \mathrm{~F}
\end{aligned}=\alpha \Delta \mathrm{T} \mathrm{a} \mathrm{Y}, ~\left(\mathrm{~T}_{2}-\mathrm{T}_{1}\right) \pi \mathrm{r}^{2} \mathrm{Y} .
$$

Here, negative sign indicates that force is inwards due to contraction of the wire.
EXAMPLE
A blacksmith fixes an iron ring on the rim of the wooden wheel of a bullock cart. The diameter of the rim and the iron ring are 5.243 m and 5.231 m respectively at $27^{\circ} \mathrm{C}$. To what temperature should the ring be heated so as to fit the rim of the wheel?

## SOLUTION

Given, $\quad T_{1}=27^{\circ} \mathrm{C}$

$$
\mathrm{L}_{\mathrm{T} 1}=5.231 \mathrm{~m}
$$

$$
\mathrm{L}_{\mathrm{T} 2}=5.243 \mathrm{~m}
$$

So,

$$
\mathrm{L}_{\mathrm{T} 2}=\mathrm{L}_{\mathrm{T} 1}\left[1+\alpha_{1}\left(\mathrm{~T}_{2}-\mathrm{T}_{1}\right)\right]
$$

$$
5.243 \mathrm{~m}=5.231 \mathrm{~m}\left[1+1.2 \times 10^{-5} \mathrm{~K}^{-1}\left(\mathrm{~T}_{2}-27^{\circ} \mathrm{C}\right)\right]
$$

$\therefore \mathrm{T}_{2}=218^{\circ} \mathrm{C}$

## EXAMPLE

A large steel wheel is to be fitted on to a shaft of the same material. At $27^{\circ} \mathrm{C}$, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm . The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range: $\alpha_{\text {steel }}=1.20 \times 10^{-5} \mathrm{~K}^{-1}$.

## SOLUTION

Here, $T_{1}=27+273=300 \mathrm{~K} ; D_{1}=8.70 \mathrm{~cm}$;

$$
D_{2}=8.69 \mathrm{~cm} \text { and } \alpha=1.20 \times 10^{-5} \mathrm{~K}^{-1}
$$

Now, $D_{2}-D_{1}=D_{1} \alpha\left(T_{2}-T_{1}\right)$

$$
\begin{aligned}
\therefore 8.69-8.70 & =8.70 \times 1.20 \times 10^{-5} \times\left(T_{2}-300\right) \\
T_{2}-300 & =-\frac{0.01}{8.70 \times 1.2 \times 10^{-5}}=-95.8 \\
T_{2} & =300-95.8=204.2 \mathrm{~K}
\end{aligned}
$$

8. SUPERFICIAL EXPANSION-


When the area of a solid increases on heating, the thermal expansion is called superficial (areal) expansion.

Suppose a metal sheet of initial surface area $\mathrm{A}_{\mathrm{o}}$ is heated through temperature $\Delta T$ and its final surface area becomes A . Let $\Delta \mathrm{A}\left(=\left(\mathrm{A}-\mathrm{A}_{0}\right)\right)$, be the change in its area.

Then
(i) $\Delta \mathrm{A} \propto \Delta \mathrm{T}$
(ii) $\Delta \mathrm{A} \propto \mathrm{A}_{0}$

Combining above factors, we get-

$$
\begin{aligned}
& \Delta A \propto A_{o} \Delta T \\
& \text { Or, } \Delta A=\beta A_{o} \Delta T \\
& \quad \therefore \beta=\frac{\Delta \mathrm{A}}{\mathrm{~A}_{\mathrm{o}} \Delta \mathrm{~T}}
\end{aligned}
$$

Here $\boldsymbol{\beta}$ is a constant called coefficient of superficial expansion and its value depends on the nature of the material.

The coefficient of superficial expansion is defined as the increase in its surface area per degree rise in its temperature.

The S.I. unit of $\beta$ is ${ }^{\circ} \mathrm{C}^{-1}$ or $\mathrm{K}^{-1}$.
The increase in the area of the sheet is $=\boldsymbol{\beta} \mathbf{x}$ original area $\mathbf{x}$ change in temperature

## EXAMPLE

Show that the coefficient of areal expansions, $(\Delta \mathrm{A} / \mathrm{A}) / \Delta T$, of a rectangular sheet of a solid is nearly twice its coefficient of linear expansion, $\alpha$. Given $\alpha \simeq \mathbf{1 0}^{\mathbf{- 5}} K^{\mathbf{1}}$.

## Solution



Fig. 11.8
Consider a rectangular sheet of the solid material of length a and breadth b (fig.11.8). When the temperature increases by $\Delta T$, a increases by $\Delta a=\alpha a \Delta T$ and $\Delta b=\alpha b \Delta T$. From fig. 11.8, the increase in area

$$
\begin{aligned}
\Delta \mathrm{A} & =\Delta \mathrm{A}_{1}+\Delta \mathrm{A}_{2}+\Delta \mathrm{A}_{3} \\
\Delta \mathrm{~A} & =\mathrm{a} \Delta \mathrm{~b}+\mathrm{b} \Delta \mathrm{a}+(\Delta \mathrm{a})(\Delta \mathrm{b}) \\
& =\mathrm{a} \alpha \mathrm{~b} \Delta \mathrm{~T}+\mathrm{b} \alpha \mathrm{a} \Delta \mathrm{~T}+(\alpha)^{2} a b(\Delta T)^{2} \\
& =\alpha \mathrm{ab} \Delta \mathrm{~T}(2+\alpha \Delta \mathrm{T})=\alpha \mathrm{A} \Delta \mathrm{~T}(2+\alpha \Delta \mathrm{T})
\end{aligned}
$$

Since $\alpha \simeq 10^{-1} \mathrm{~K}^{-1}$, the product $\alpha \Delta \mathrm{T}$ for small changes in temperature is small in comparison with 2 and may be neglected.

Hence,

$$
\left(\frac{\Delta \mathrm{A}}{\mathrm{~A}}\right) \frac{1}{\Delta \mathrm{~T}}=2 \alpha \simeq \beta
$$

## EXAMPLE

A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at $27.0^{\circ} \mathrm{C}$.
What is the change in the diameter of the hole when the sheet is heated to $227^{\circ} \mathrm{C}$ ?
Coefficient of linear expansion of copper $=1.70 \times 10^{-5} \mathrm{~K}^{-1}$.

## SOLUTION

## https://www.efoto.lt/files/images/13773/SDC10003.preview.JPG

## Given:

$$
\begin{aligned}
& \Delta \mathrm{T}=227.0-27.0=200.0^{\circ} \mathrm{C} ; \\
& \alpha=1.70 \times 10^{-5} \mathrm{~K}^{-1}
\end{aligned}
$$

Therefore, coefficient of superficial expansion of copper,

$$
\beta=2 \alpha=3.40 \times 10^{-5 \circ} C^{-1}
$$

Diameter of hole at $27.0^{\circ} \mathrm{C}, d_{1}=4.24 \mathrm{~cm}$
Therefore, surface area of hole at $27.0^{\circ} \mathrm{C}$,

$$
S_{1}=\frac{\pi d_{1}^{2}}{4}=\frac{\pi}{4} \times(4.24)^{2}=\pi \times 4.4944 \mathrm{~cm}^{2}
$$

Let $S_{2}$ be the surface area of the hole at $227^{\circ} \mathrm{C}$, then,

$$
\begin{aligned}
S_{2} & =S_{1}(1+\beta \Delta T) \\
& =\pi \times 4.4944 \times\left(1+3.40 \times 10^{-5} \times 200\right) \\
& =\pi \times 4.4944 \times 1.0068=\pi \times 4,525 \mathrm{~cm}^{2}
\end{aligned}
$$

If $\mathrm{d}_{2}$ is diameter of the hole at $227^{\circ} \mathrm{C}$,
then

$$
S_{2}=\frac{\pi d_{2}^{2}}{4}
$$

After comparing above equations

$$
\begin{gathered}
\frac{\pi \mathrm{d}_{2}^{2}}{4}=\pi \times 4,525 \\
\mathrm{~d}_{2}=(4 \times 4.525)^{1 / 2}=4.2544 \mathrm{~cm}
\end{gathered}
$$

Therefore, change in diameter of the hole,

$$
d_{2}-d_{1}=4.2544-4.24=0.0144 \mathrm{~cm}
$$

## 9. VOLUMETRIC EXPANSION-

In general, it is the overall volume of a solid that changes when its temperature is changed. However, we can talk of
(i) Change in its length when its length is much more than its other two dimensions.
(ii) Change in its area when one of its dimensions, say, the thickness, is very small in comparison to its other two dimensions. When all the three dimensions of a solid are comparable to one another, we need to talk of changes in its volume.

Hence, in bulk solid of any shape the expansion occurs in volume. For liquids and gases, we, of course, need to talk of volume expansion only.

When the volume of a material increases on heating, the thermal expansion is called volume expansion or cubical expansion.

Suppose a solid block of initial volume $\mathrm{V}_{0}$ is heated through a temperature $\Delta T$ and its final volume is V . Then, the change in its volume, $\Delta \mathrm{V}\left(=\left(\mathrm{V}-\mathrm{V}_{\mathrm{o}}\right)\right)$
(i) $\Delta V \propto \Delta T$
(ii) $\Delta V \propto V_{o}$

Combining above factors, we get-

$$
\begin{aligned}
& \Delta V \propto V_{o} \Delta T \\
& \text { Or, } \Delta V=\gamma V_{o} \Delta T \\
& \quad \gamma=\frac{\Delta V}{V_{o} \Delta T}
\end{aligned}
$$

Here $\quad \gamma \quad$ is a constant called coefficient of cubical expansion.
Coefficient of volume expansion can also be represented by $\alpha_{V}$.
The coefficient of cubical expansion of a substance is defined as the increase in volume per unit original volume per degree rise in its temperature.

The unit of $\gamma$ is ${ }^{0} \mathrm{C}^{-1}$ or $\mathrm{K}^{-1}$.

Interestingly $\alpha, \beta$ and $\gamma$ all have the same unit but imply different expansions. The three are related as follows:

$$
\gamma \simeq 3 \alpha
$$

And,

$$
\beta \simeq 2 \alpha
$$

$$
\therefore \quad \alpha: \beta: \gamma=1: 2: 3
$$

The above is a simple relation between the coefficient of volume expansion $\gamma$ and coefficient of linear expansion $\alpha$, can be obtained as follows:

Imagine a cube of length, L , that expands equally in all directions when its temperature increases by $\Delta \mathrm{T}$.


We have $\Delta \mathrm{L}=\alpha \mathrm{L} \Delta \mathrm{T}$ so, $\Delta \mathrm{V}=(\mathrm{L}+\Delta \mathrm{L})^{3}-\mathrm{L}^{3} \simeq 3 \mathrm{~L}^{2} \Delta \mathrm{~L}=3 \mathrm{Al}^{3} \Delta \mathrm{~T}=3 \alpha \mathrm{~V}_{\mathrm{o}} \Delta \mathrm{T}$ (terms in $(\Delta \mathrm{L})^{2}$ and $(\Delta \mathrm{L})^{3}$ have been neglected since $\Delta \mathrm{L}$ is small compared to L )

This gives $\gamma=3 \boldsymbol{\alpha}$


Here $\gamma$ is also a characteristic of the substance but is not strictly a constant. In general, it depends on temperature. It is observed that $\gamma$ becomes constant only at high temperatures

Values of coefficient of volume expansion for some substances are given below.

|  |  |
| :---: | :---: |
| Materials | $\gamma\left(\mathrm{K}^{-1}\right)$ |
|  |  |
| Aluminium | $7 \times 10^{-5}$ |
| Brass | $6 \times 10^{-5}$ |
| Iron | $3.55 \times 10^{-5}$ |
| Paraffin | $58.8 \times 10^{-5}$ |
| Glass(ordinary) | $2.5 \times 10^{-5}$ |
| Glass(pyrex) | $1 \times 10^{-5}$ |
| Hard rubber | $2.4 \times 10^{-4}$ |
| Invar | $2 \times 10^{-6}$ |
| Mercury | $18.2 \times 10^{-5}$ |
| Water | $20.7 \times 10^{-5}$ |
| Alcohol (ethyl) | $110 \times 10^{-5}$ |

The table gives the values of co-efficient of volume expansion of some common substances in the temperature range $0-100{ }^{\circ} \mathrm{C}$.

You can see that thermal expansion of these substances (solids and liquids) is rather small, with materials like pyrex glass and invar (a special iron-nickel alloy) having particularly low values of gamma. From this Table we find that the value of coefficient of volume expansion for alcohol (ethyl) is more than mercury; alcohol expands more than mercury for the same rise in temperature.

Common solid materials used in engineering, usually have coefficients of thermal expansion that do not vary significantly over the range of temperatures where they are designed to be used. Hence, when extremely high accuracy is not required, practical calculations can be based on a constant average, value of their coefficient of expansion

## EXAMPLE

The coefficient of volume expansion of glycerine is $49 \times 10^{-5} \mathrm{~K}^{-1}$. What is the fractional change in its density for a $30^{\circ} \mathrm{C}$ rise in temperature?

## SOLUTION

Given: $\gamma=49 \times 10^{-5} K^{-1} ; \Delta T=30^{\circ} \mathrm{C}$

Let there be m grams of glycerine and its initial volume be V . suppose that the volume of the glycerine becomes $V^{\prime}$ after a rise of temperature of $30^{\circ} \mathrm{C}$. Then,
$\mathrm{V}^{\prime}=\mathrm{V}+(1+\gamma \Delta \mathrm{T})=\mathrm{V}\left(1+49 \times 10^{-5} \times 30\right)=1.0147 \mathrm{~V}$
Initial density of the glycerine, $\rho=\frac{m}{V}$
Final density of the glycerine,
$\rho^{\prime}=\frac{m}{V^{\prime}}=\frac{m}{1.0147 V}=\frac{\rho}{1.0147}=0.9855 \rho$

Therefore, fractional change in the value of density of glycerine,

$$
\frac{\rho^{\prime}-\rho}{\rho}=\frac{\rho-0.9855 \rho}{\rho}=0.0145
$$

## CONSIDER THESE

- What causes expansion due to heat?
- Why should coefficients of linear, superficial and volume expansion be related for the same material? Would the value of $\alpha, \beta$ and $\gamma$ be the same even if the shape of material changes?
- Would coefficient of linear expansion for a copper wire, copper rod, copper strip be the same?


## 10. THERMAL STRESS

It is the force set up per unit cross-sectional area of a body when its thermal expansion or contraction is resisted.

What happens by preventing the thermal expansion of a rod by fixing its ends rigidly? Clearly, the rod acquires a compressive strain due to the external forces provided by the rigid support at the ends. The corresponding stress set up in the rod is called thermal stress.

Imagine a rod fixed between two sections of the wall. The rod develops a stress within, if the temperature change causes a change in its length. Let the length of the rod be L, change in temperature $\Delta T$, and let the coefficient of linear expansion of its material be $\alpha$

Now,

$$
\mathrm{Y}=\frac{\text { stress }}{\text { strain }}
$$

Also a

$$
\begin{aligned}
& \text { longitudinal strain }=\frac{\Delta L}{L}=\alpha \Delta T \\
\therefore & \text { Stress(= thermal stress) }=Y \alpha \Delta T
\end{aligned}
$$

If A is the area of cross section of the rod under consideration the force developed is

$$
\text { stress } \times \mathbf{A}=\mathbf{Y} \alpha \Delta T A=\frac{Y \Delta L A}{L}
$$

## EXAMPLE

A steel rail, of length 5 m and area of cross section $40 \mathrm{~cm}^{2}$ that is prevented from expanding while the temperature rises by $10^{\circ} \mathrm{C}$. The coefficient of linear expansion of steel is $\alpha$ (steel) $=1.2 \times 10^{-5} \mathrm{~K}^{-1}$.

Calculate the external force developed. (Given: $Y_{\text {steel }}=\mathbf{2 \times 1 0} \mathbf{N m}^{\mathbf{1 1}} \mathbf{N m}^{-2}$ )

## SOLUTION

The compressive strain $\frac{\Delta L}{L}=\alpha \Delta T=1.2 \times 10^{-5} \times 10=1.2 \times 10^{-4}$
Young's modulus of steel $=2 \times 10^{11} \mathrm{Nm}^{-2}$
Therefore the thermal stress developed is

$$
\frac{\Delta \mathrm{F}}{\mathrm{~A}}=\mathrm{Y}\left(\frac{\Delta \mathrm{~L}}{\mathrm{~L}}\right)=2.4 \times 10^{7} \mathrm{Nm}^{-2}
$$

which corresponds to an external force, on the 'hold', of

$$
\mathrm{F}=\mathrm{AY}\left(\frac{\Delta \mathrm{~L}}{\mathrm{~L}}\right)=2.4 \times 10^{7} \times 40 \times 10^{-4} \mathrm{~N}=\mathbf{9 . 6} \times \mathbf{1 0}^{4} \mathrm{~N}
$$

If two such steel rails, fixed at their outer ends, are in contact at their inner ends, a force of this magnitude can easily bend the rails.

## EXAMPLE

A brass rod, of length 50 cm and diameter 3.0 mm , is joined to a steel rod of the same length and diameter.

What is the change in length of the combined rod at $250{ }^{\circ} \mathrm{C}$, if the original lengths are at $40.0^{\circ} \mathrm{C}$ ?

Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand $\left(\right.$ Co-efficient of linear expansion of brass $=2.0 \times 10^{-5} \mathrm{~K}^{-1}$, steel $\left.=1.2 \times 10^{-5} \mathrm{~K}^{-1}\right)$.

## SOLUTION

Given: length of brass and steel rod $=50 \mathrm{~cm}$ i.e. $\mathrm{L}_{1}=\mathrm{L}_{2}=50 \mathrm{~cm}$

$$
\begin{aligned}
& \text { Radius of brass and steel rod, } \mathrm{r}_{1}=\mathrm{r}_{2}=1.5 \mathrm{~mm}=1.5 \times 10^{-3} \mathrm{~m} \\
& \text { Change in length of brass rod at } 250^{\circ} \mathrm{C}=\mathrm{L}_{1} \alpha_{1} \Delta \mathrm{~T} \\
& \\
& =50 \times 2.0 \times 10^{-5} \times(250-40)=0.2205 \mathrm{~cm}
\end{aligned}
$$

Change in length of steel rod at $250^{\circ} \mathrm{C}=\mathrm{L}_{2} \alpha_{2} \Delta \mathrm{~T}$

$$
=50 \times 1.2 \times 10^{-5} \times(250-40)=0.126 \mathrm{~cm}
$$

Change in length of combined rod $=\Delta \mathrm{L}_{1}+\Delta \mathrm{L}_{2}$

$$
=0.220+0.126=0.346 \mathrm{~cm}
$$

## CONSIDER THESE

- Since every-body has a tendency of resisting the change in its volume, shape and position; is there any property of the body which opposes the change in volume due to thermal expansion?
- If we fix the volume of liquid and increase its temperature would there be thermal stress?


## 11. THERMAL EXPANSION IN LIQUIDS-

The expansion of liquids is has to be measured by having them in a container. When a liquid expands in a vessel, the vessel expands along with the liquid. Hence the observed increase in volume of the liquid level is not the actual increase in its volume. The expansion of the liquid, relative to the container is called as Apparent Expansion. The actual expansion of the liquid is called Real (or) Absolute expansion.

The ratio of the apparent increase in volume of the liquid, to the original volume, per unit rise of temperature is known as its coefficient of apparent expansion.

For small and same rise in temperature, the increase in volume (real expansion) of liquid is equal to the sum of apparent increase in volume (apparent expansion) of liquid and the increase in volume of the containing vessel. Thus a liquid has two coefficients of expansion.


## http://images.tutorvista.com/content/heat/liquid-expansion.jpeg

When measuring the expansion of a liquid, the measurement must account for the expansion of the container as well. Let a liquid be put in a flask that has been constructed with a long narrow stem. Let the flask be filled with enough liquid so that the stem itself is partially filled. When placed in a heat bath, it would be observed the column of liquid in the stem first drops; this is followed by the immediate increase of that column. When the flask-liquid-heat bath system is at the same temperature, the liquid column in the stem, is at a level higher than its initial value.

The initial observation of the column of liquid dropping is not due to an initial contraction of the liquid but rather the expansion of the flask as it contacts the heat bath first.

Soon after, the liquid in the flask is heated by the flask itself and begins to expand.
Since liquids typically have a greater expansion over solids, the expansion of the liquid in the flask eventually exceeds that of the flask, causing the column of liquid in the flask to rise. A direct measurement of the increase in height of the liquid column would provide a measurement of the apparent expansion of the liquid.

The absolute expansion of the liquid is the apparent expansion corrected for the expansion of the containing vessel.

- Liquids do not have linear and superficial expansion; they only have volumetric expansion.
- Since liquids are always to be heated along with a vessel (which contains them) initially, on heating the system (liquid + vessel), the level of liquid in vessel falls (as it is the vessel that initially expands since it absorbs heat first. Later on, the level of the liquid starts rising due to a greater expansion of the liquid.
- The actual increase in the volume of the liquid = The apparent increase in the volume of the liquid + the increase in the volume of the vessel
Liquids have two coefficients of volume expansion.
(i) Co-efficient of apparent expansion
(ii) Co-efficient of real expansion
- Density of liquid (usually)decreases with rise in temperature
- A liquid may become more flowy with increase in temperature.


## CONSIDER THESE

- What is that limit up to which a liquid can expand without converting in to vapour?
- Why people suffer most of the heart attacks in the winter seasons?


## 12. ANOMALOUS EXPANSION OF WATER -

Almost all liquids expand on heating, but water behaves in a peculiar manner.
When water at 0 C is heated, its volume decreases and therefore the density increases. The water at $4^{\circ} \mathrm{C}$ has the maximum density.

Water contracts when heated from $0^{\circ} \mathrm{C}$ to $4^{\circ} \mathrm{C}$; it then expands when heated from $4^{\circ} \mathrm{C}$ to $100^{\circ} \mathrm{C}$.
Remembering that density is mass divided by volume, we can now understand why water, at 4 ${ }^{\circ} \mathrm{C}$ is denser than water below and above $4^{\circ} \mathrm{C}$. This also explains why lakes freeze on the top first and not throughout. As the temperature of water on the top of the lake drops to $4^{\circ} \mathrm{C}$, it becomes denser than the water below it. Thus it sinks to the bottom, allowing the warmer water to rise up to the top and again cool down to $4^{\circ} \mathrm{C}$.

Only when the entirety of the lake is at $4^{\circ} \mathrm{C}$, the lake can start to freeze. It freezes from the top down, because water, below $4^{\circ} \mathrm{C}$, is less dense than water at $4^{\circ} \mathrm{C}$.

(a)


Thermal expansion of water (NCERT) the two graphs volume vs temperature and density versus temperature

Water has maximum density at $4{ }^{0} \mathrm{C}$, volume of 1 kg water is the least at $4{ }^{0} \mathrm{C}$

- Water exhibits an anomalous behaviour; anomalous means deviating from or inconsistent with the common behaviour, form, or rule; irregular; abnormal
- It contracts on heating between $0^{\circ} \mathrm{C}$ and $4^{\circ} \mathrm{C}$.
- The volume of a given amount of water decreases as it is cooled from room temperature, until its temperature reaches $4^{\circ} \mathrm{C}$,
- Below $4{ }^{\circ} \mathrm{C}$, the volume increases, and therefore the density decreases
- This means that water has a maximum density at $4^{\circ} \mathrm{C}$.

This property has an important environmental effect: Bodies of water, such as lakes and ponds, freeze at the top first.
As a lake cools toward $4^{\circ} \mathrm{C}$, water near the surface loses energy to the atmosphere, becomes denser, and sinks; the warmer, less dense water, near the bottom rises. However, once the colder water on top reaches temperature below $4^{\circ} \mathrm{C}$, it becomes less dense and remains at the surface, where it freezes. If water did not have this property, lakes and ponds would freeze from the bottom up, which would destroy much of their animal and plant life.


Now we can explain why a lake freezes at the top first, rather than throughout or from the bottom first?

Do you think the same happens to ice cubes in the freezer compartment of our home refrigerators?

Try and see.

## 13. EXPANSION OF GASES-

Gases, at ordinary temperature, expand more than solids and liquids. For gases, the coefficient of volume expansion is dependent on temperature.

## For an ideal gas-

the Volumetric expansion coefficient of the gas. It is the rate of change of volume of gas with respect to the change in temperature, if the gas expands under constant pressure.

As can be imagined gases at ordinary temperature expand more than solids and liquids. For liquids, the coefficient of volume expansion is relatively independent of the temperature.

However, for gases it is dependent on temperature.
For an ideal gas, the coefficient of volume expansion, at constant pressure, can be found from the ideal gas equation:
$P V=\mu R T$
At constant pressure, we have

$$
\mathrm{P} \Delta \mathrm{~V}=\mu \mathrm{R} \Delta \mathrm{~T}
$$

$$
\begin{gathered}
\therefore \frac{\Delta V}{V}=\frac{\Delta T}{T} \\
\gamma=\frac{\Delta V}{V \cdot \Delta T} \\
\gamma=\frac{1}{T}
\end{gathered}
$$

- $\quad \gamma$ for gases is much larger than that for solids and liquids
- $\gamma=\frac{1}{T}$ shows the temperature dependence of $\gamma$; it decreases with increasing temperature.
- For a gas at room temperature $\left(\simeq 27^{\circ} C\right)$ and constant pressure , $\gamma$ is about $\mathbf{3 . 3} \times \mathbf{1 0}^{-\mathbf{3}} \mathbf{K}^{\mathbf{- 1}}$.

The value of $\boldsymbol{\gamma}$ depends on temperature?
Yes it does depend on temperature.

$$
\mathbf{P V}=\mathbf{n} \mathbf{R T}
$$

So

$$
V \propto T
$$

Or
Volume is proportional to temperature in Kelvin.
To go from 0 degrees Celsius to 1 degree Celsius the temperature in Kelvin changes from 273 K to 274 K (approximately)

So the volume changes from
$\mathbf{V}$ to $\frac{274}{273} \mathbf{V}$ which is equal to $\mathbf{V}+\frac{1}{273} \mathbf{V}$ - so there is an increase of $\frac{1}{273} \mathbf{V}$
Note in this rise of $\frac{1}{273} \mathbf{V}$
The 273 comes from the initial temperature of $273 K$ so that if the temperature were say 100 Celsius or 373 K the factor would be $\frac{1}{373} \mathbf{V}$

## EXAMPLE

A large steel wheel is to be fitted on to a shaft of the same material. At $27^{\circ} \mathrm{C}$, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm . The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range: $\alpha_{\text {steel }}=1.20 \times 10^{-5} \mathrm{~K}^{-1}$.

## SOLUTION

Here, $T_{1}=27+273=300 \mathrm{~K} ; D_{1}=8.70 \mathrm{~cm} ;$

$$
D_{2}=8.69 \mathrm{~cm} \text { and } \alpha=1.20 \times 10^{-5} \mathrm{~K}^{-1}
$$

Now, $D_{2}-D_{1}=D_{1} \alpha\left(T_{2}-T_{1}\right)$

$$
\begin{gathered}
\therefore 8.69-8.70=8.70 \times 1.20 \times 10^{-5} \times\left(T_{2}-300\right) \\
T_{2}-300=-\frac{0.01}{8.70 \times 1.2 \times 10^{-5}}=-95.8
\end{gathered}
$$

$$
T_{2}=300-95.8=204.2 \mathrm{~K}
$$

## EXAMPLE

A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at $27.0^{\circ} \mathrm{C}$.
What is the change in the diameter of the hole when the sheet is heated to $227^{\circ} \mathrm{C}$ ?
Coefficient of linear expansion of copper $=1.70 \times 10^{-5} \mathrm{~K}^{\mathbf{- 1}}$.

## SOLUTION

Given: $\Delta T=227.0-27.0=200.0^{\circ} \mathrm{C} ; \alpha=1.70 \times 10^{-5} \mathrm{~K}^{-1}$
Therefore, coefficient of superficial expansion of copper,

$$
\beta=2 \alpha=3.40 \times 10^{-5 \circ} C^{-1}
$$

Diameter of hole at $27.0^{\circ} \mathrm{C}, d_{1}=4.24 \mathrm{~cm}$
Therefore, surface area of hole at $27.0^{\circ} \mathrm{C}$,

$$
S_{1}=\frac{\pi d_{1}^{2}}{4}=\frac{\pi}{4} \times(4.24)^{2}=\pi \times 4.4944 \mathrm{~cm}^{2}
$$

Let $S_{2}$ be the surface area of the hole at $227^{\circ} \mathrm{C}$, then,
$S_{2}=S_{1}(1+\beta \Delta T)$

$$
\begin{aligned}
& =\pi \times 4.4944 \times\left(1+3.40 \times 10^{-5} \times 200\right) \\
& =\pi \times 4.4944 \times 1.0068=\pi \times 4,525 \mathrm{~cm}^{2}
\end{aligned}
$$

If $d_{2}$ is diameter of the hole at $227^{\circ} \mathrm{C}$, then

$$
S_{2}=\frac{\pi d_{2}^{2}}{4}
$$

After comparing above equations

$$
\begin{gathered}
\frac{\pi d_{2}^{2}}{4}=\pi \times 4,525 \\
d_{2}=(4 \times 4.525)^{1 / 2}=4.2544 \mathrm{~cm}
\end{gathered}
$$

Therefore, change in diameter of the hole,

$$
\mathrm{d}_{2}-\mathrm{d}_{1}=4.2544-4.24=0.0144 \mathrm{~cm}
$$

## EXAMPLE

The coefficient of volume expansion of glycerine is $49 \times 10^{-5} \mathrm{~K}^{-1}$. What is the fractional change in its density for a $30{ }^{\circ} \mathrm{C}$ rise in temperature?

## SOLUTION

Given: $\gamma=49 \times 10^{-5} K^{-1} ; \Delta T=30^{\circ} \mathrm{C}$
Let there be m grams of glycerine and its initial volume be V . suppose that the volume of the glycerine becomes V' after a rise of temperature of $30^{\circ} \mathrm{C}$. Then,
$\mathrm{V}^{\prime}=\mathrm{V}+(1+\gamma \Delta \mathrm{T})=\mathrm{V}\left(1+49 \times 10^{-5} \times 30\right)=1.0147 \mathrm{~V}$
Initial density of the glycerine, $\rho=\frac{m}{V}$
Final density of the glycerine,
$\rho^{\prime}=\frac{m}{V^{\prime}}=\frac{m}{1.0147 V}=\frac{\rho}{1.0147}=0.9855 \rho$
Therefore, fractional change in the value of density of glycerine,
$\frac{\rho^{\prime}-\rho}{\rho}=\frac{\rho-0.9855 \rho}{\rho}=0.0145$

```
=10}\times1\mp@subsup{0}{}{3}=1\mp@subsup{0}{}{4}
```


## APPLICATION OF THERMAL EXPANSION-

The expansion and contraction of materials must be considered when designing large structures, when using tape or chain to measure distances for land surveys, when designing molds for casting hot material, and in all other such engineering applications where large changes in dimension due to temperature, are expected.

Thermal expansion is also used in mechanical applications to fit parts over one another, e.g. a bushing can be fitted over a shaft by making its inner diameter slightly smaller than the diameter of the shaft, then heating it until it fits over the shaft, and allowing it to cool after it has been pushed over the shaft, thus achieving a 'shrink fit'. Induction shrink fitting is a common industrial method to pre-heat metal components between $150{ }^{\circ} \mathrm{C}$ and $300^{\circ} \mathrm{C}$ thereby causing them to expand and allow for the insertion or removal of another component.

There exist some alloys with a very small linear expansion coefficient; they are used in applications that demand very small changes in physical dimension over a range of temperatures. One of these is Invar 36, with $\alpha$ approximately equal to $0.6 \times 10^{-6} \mathrm{~K}^{-1}$.

These alloys are useful in aerospace applications where wide temperature swings may occur.
Precision engineering nearly always requires the engineer to pay attention to the thermal expansion of the product. For example, when using a scanning electron microscope, even small changes in temperature, such as 1 degree, can cause a sample to change its position relative to the focus point.

We can list the following as examples of situations where thermal expansion has to be taken into consideration:
(i) The transmission cables are not tightly fixed to the poles.
(ii) A small gap is left between the iron rails of railway tracks.
(iii) Bridges, and roofs of houses, using girders, are placed on rollers to withstand variation in temperature throughout the year.
(iv)Stoppers, jammed at the mouth of bottles, can be taken out by heating.
(v)Metal rings are mounted on wooded barrels by heating.
(vi)Pendulum clocks have invar pendulums as ordinary metal pendulums would change in length and hence give seasonal time variation.
(vii) Thermometers are another application of thermal expansion - most contain a liquid (usually mercury or alcohol) which is constrained to flow in only one direction (along the tube) due to changes in volume brought about by changes in temperature.
(vii) Bimetallic strip - made of two strips of dissimilar metal riveted together, which bends due to unequal expansion due to same temperature changes.it is widely used as thermostat.

## EXAMPLE

You have been given bars of identical dimensions of following metals/materials along with their $\alpha$-values, for making a bimetallic strip:
Aluminium ( $\alpha=23 \times 10^{-6} \mathrm{~K}^{-1}$ );
Nickel ( $\alpha=13 \times 10^{-6} \mathrm{~K}^{-1}$ )
Copper ( $\alpha=17 \times 10^{-6} \mathrm{~K}^{-1}$ );
Invar $\left(\alpha=0.9 \times 10^{-6} \mathrm{~K}^{-1}\right)$
Iron ( $\alpha=12 \times 10^{-6} \mathrm{~K}^{-1}$ );
$\operatorname{Brass}\left(\alpha=18 \times 10^{-6} \mathrm{~K}^{-1}\right)$
which pair of metals/materials would you select as best choice for making a bimetallic strip for pronounced effect of bending? Why?

## SOLUTION

Though the order of magnitude for each coefficient of expansion is the same
The best combination would be aluminum and invar

## EXAMPLE

What would happen if the strip is heated to a high temperature? and $\backslash n a m e$ a few devices in which bi-metallic strips are generally used as a thermostat

## SOLUTION

This is would curl
Thermometers dial type
Fire alarm, toasters, sandwich toasters, electric grills, food warmers thermostats in fridge, geysers, MCB (mini circuit breakers, electric irons, ovens , microwave ovens, heaters ,air conditioners etc .

## 14. ACTIVITY USING BI-METALLIC STRIP <br> (Activity included as part of course work)

To observe and explain the effect of heating on a bi-metallic strip

## APPARATUS AND MATERIAL REQUIRED

A iron-brass bi-metallic strip with an insulating (wooden) handle; heater/burner.

## DESCRIPTION OF THE DEVICE

A bi-metallic strip is made of two bars/strips of different metals (materials), but of same dimensions. These metallic bars/strips (A and B) are put together lengthwise and firmly riveted. An insulating (wooden) handle is also fixed at one end of the bi-metallic strip. A bi-metallic strip can be made by selecting metals (materials) with widely different values of coefficients of linear thermal expansion. The bi-metallic strip is designed to be straight at room temperature, . When the bi-metallic strip is heated, both metallic pieces expand to different extents because of their different coefficients of linear expansion, As a result, the bimetallic strip appears to bend when heated.

## PRINCIPLE

The linear thermal expansion is the change in length of a bar on heating. If $L_{1}$ and $L_{2}$ are the lengths of rod/bar of a metal at temperatures $t_{1}{ }^{\circ} \mathrm{C}$ and $\mathrm{t}_{2}{ }^{\circ} \mathrm{C}$ (such that $\mathrm{t}_{2}>\mathrm{t}_{1}$ ), the change in length $\left(L_{2}-L_{1}\right)$ is directly proportional to the original length $L_{1}$ and the rise in temperature $\left(\mathrm{t}_{2}-\mathrm{t}_{1}\right)$.
Then,
$\left(\mathrm{L}_{2}-\mathrm{L}_{1}\right)=\alpha \mathrm{L}_{1}\left(\mathrm{t}_{2}-\mathrm{t}_{1}\right)$ or $\mathrm{L}_{2}=\mathrm{L}_{1}\left[1+\alpha\left(\mathrm{t}_{2}-\mathrm{t}_{1}\right)\right]$ and
$\alpha=\left(\mathrm{L}_{2}-\mathrm{L}_{1}\right) /\left(\mathrm{t}_{2}-\mathrm{t}_{1}\right)$
Where $\alpha$ is the coefficient of linear thermal expansion of the material of the bar/rod.
The coefficient of linear thermal expansion $(\alpha)$ is the increase in length per unit length for unit degree rise in temperature of the bar.
It is the property of the material of which the bar /rod is made of .It is expressed in SI units as $\mathbf{K}^{-1}$.

## PROCEDURE

(i) Light a burner or switch on the electric heater.
(ii) Keep the bi-metallic strip in the horizontal position by holding it with the insulated handle and heat it with the help of burner/ heater. Note which side of the bi-metallic strip is in direct contact with the heat source.
(iii)Observe the effect of heating the strip. Note carefully the direction of the bending of the free end of the bi-metallic strip, whether it is upwards or downwards?
(iv)Identify the metal (A or B) which is on the convex side of the bi-metallic strip and also the one which is on its concave side. Which one of the two metals/materials strips has a larger thermal expansion? (The one on the convex side of the bimetallic strip will expand more and hence have larger linear thermal expansion).
(v) Note down the known values of coefficient of linear thermal expansion of two metals ( $A$ and B) of the bi-metallic strip. Verify whether the direction of bending (upward or downward) is on the side of the metal/material having lower coefficient of linear thermal expansion.
(vi) Take the bi-metallic strip away from the heat source. Allow the strip to cool to room temperature.
(vii) Repeat the Steps 1 to 6 to heat the other side of the bi-metallic strip. Observe the direction of bending of the bi-metallic strip. What change, if any, do you observe in the direction of bending of the strip in this case relative to that observed earlier in Step 3?

## RESULT

The bending of a bi-metallic strip on heating is due to difference in coefficient of linear expansion of the two metals of the strip.

## PRECAUTIONS

The two bars (strips) should be firmly riveted near their ends.

## DISCUSSION

The direction of bending of the bi-metallic strip is towards the side of the metal which has lower value of linear thermal expansion.

https://www.youtube.com/watch?v=xeRZmNWWGlc

This wonderful kit has been designed by LG of Korea. The heart of the Fire Alarm is a bimetallic strip. When heated the strip expands completing an electric circuit. This switches ON a Fire Alarm and the LED warning light starts to blink signalling danger!

## 15. SUMMARY

- Materials expand on heating
- Gases expand more than liquids and solids
- Coefficient of linear expansion is the change in length per unit length per unit rise in temperature
- Coefficient of superficial expansion is the change in surface area per unit area per unit rise in temperature
- Coefficient of volume expansion is the change in volume per unit volume per unit rise in temperature
- Liquids are contained in vessels and the thermal expansion of vessel should be taken into account for knowing about their real expansion.
- Expansion in gases can take place by change in temperature, change in pressure or both . the coefficient of volume expansion of gases is described at constant pressure is large;
- $\gamma=1 / \mathrm{T}$ for one mole of ideal gas the coefficient of expansion for all gases is the same at constant pressure
- Engineers have to account for thermal expansion in designs for home and other applications.

